A Consideration of the Comparative Cost Model Using Three-Dimensional Diagrams

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Abstract: This study uses three-dimensional diagrams to express the export function, total labor demand function of individual countries, and the world-market supply function, and describes a method to specify the trade equilibrium point. The trade model is constructed as a multi-commodity, multi-country general equilibrium model, but actual simulation is conducted by a two-country, two-commodity model. The specific factors model is adopted when assuming the production function has a decreasing return to scale and only one variable (i.e., labor input), and wage is assumed to be an exogenous variable. First, the total labor demands of each country are derived as functions of the price of each good, and these total labor demand functions are expressed by three-dimensional diagrams. Second, the "price possibility frontier" domain for each country is defined by the total labor demand function and labor endowment. Next, the equilibrium trade point is specified by the "price possibility frontier" and the restriction on the balance of supply and demand of each good. Last, the normative equilibrium trade point is simulated, using actual data from the Japan–U.S. international input–output table for the year 2000. The simulated normative trade follows the same direction as actual trade. This result suggests the usefulness of this method for analyzing how the internalization of public benefits from forestry or agriculture impacts the pattern of international specialization.

1. Introduction

The goal of this study is to investigate the impact of the internalization of public benefits from forestry on the pattern of international specialization, using a multi-

commodity, multi-country comparative cost model, including intermediate goods as a linear programming problem. Improvements were necessary because the original version of this model does not consider price (Ejiri, 2005). As a preliminary step toward such improvement, this paper presents a method to reveal the process leading to trade equilibrium; the method involves three-dimensional diagrams devised by the author, as well as export and other functions.

Only two countries, Japan and the U.S., are considered here. Two sectors (goods), agriculture (wheat) and manufacturing (cloth), are considered in the first Ricardian model and in the following example using the specific factors model. In the last simulation, which includes actual data, two aggregated sectors are considered. The specific factors model is used as the trade model.

The specific factors model describes the production function as follows (Kimura, 2000):

Agriculture: $f(K_a, L)$,

(L : labor force, K_a : factors particular to agriculture such as natural resources)

Manufacturing: $f(K_m, L)$,

(L : labor force, K_m : factors particular to manufacturing such as capital equipment)

In the specific factors model, L in the production functions is supposed to be mobile within a country but not between countries. The variables K_a and K_m are considered immobile both between and within countries. Here, K_a and K_m are supposed to be constant during the entire specialization period. Under this assumption, the production function $f(K_a, L)$ can be written as $f_{Ka}(L)$, and $f(K_m, L)$ as $f_{Km}(L)$. For simplicity, $f_{Ka}(L)$ and $f_{Km}(L)$ are written as $f(L)$ hereafter.

There is some evidence that differences in specialization patterns may be explained by the difference in labor productivity alone (Krugman 2003). The correlation between labor productivity and export and import rates is also examined here (Japan Productivity Center for Socio Economic Development, 2001; Management and Coordination Agency, Government of Japan, 1999; Ministry of International Trade and Industry, 2000; OECD, 2002; U.S. Bureau of the Census,

1996).   Figure 1 shows the correlation between q_r (Japan/U.S. relative labor productivity) and e_r (Japan/U.S. relative export rate), and between q_r and m_r (import rate), with respect to 11 goods (sectors) chosen from the 13 tradable goods listed in the Japan–U.S. international input–output table for 1995.   The excluded goods are "textiles" and "petroleum products" (Note 1).
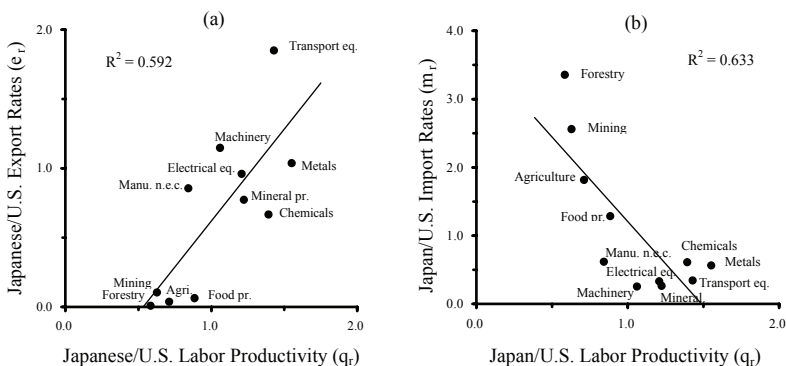

Figure 1. Correlation between Japan/U.S. labor productivity and Japan/U.S. export and import rates

Note: $q_r \equiv q_J / q_A$ (Japan/U.S. labor productivity), where, $q_J \equiv G_J^o / L_J^o$, $q_A \equiv G_A^o / L_A^o$,

$e_r \equiv e_J / e_A$ (Japan/U.S. export rate), where , $e_J \equiv E_J^o / X_J^o$, $e_A \equiv E_A^o / X_A^o$

$m_r \equiv m_J / m_A$ (Japan/U.S. import rate), where , $m_J \equiv M_J^o / D_J^o$, $m_A \equiv M_A^o / D_A^o$

(L_J^o , G_J^o , X_J^o , E_J^o , M_J^o , D_J^o : employees, gross value added, output, export, import, and total domestic demand for each sector or good in Japan in 1995, respectively.

L_A^o , G_A^o , X_A^o , E_A^o , M_A^o , D_A^o : The same in the U.S.)

Although the capital intensities differ among the sectors, Fig. 1 (a), (b) shows fairly high correlations between the variables. Furthermore, although the plots are limited and the two sectors mentioned above are excluded, the figure suggests the validity of explaining differences in the specialization pattern using only the difference in labor productivity.

In this study, the "flow approach" principle is used to decide the exchange rate. Because foreign currency supply and demand are supposed to occur only through the export or import of merchandise, and because of the capital account, foreign currency reserves are assumed to be constant, and the exchange rate is decided solely so as to maintain the trade balance.

Hereafter, "present" or "before specialization" refer to stable state economies before trade, and "future" or "after specialization" indicate the stable state economies after trade. Additionally, "world" indicates the total market consisting only of Japan and the U.S.

2. Ricardian Model

Here, the Ricardian model is explained as the foundation of the specific factors model, using simple parameters. Table 1 (1) and (2) show the outputs and allocated labor forces of each sector in each country, respectively. Table 1 (3) shows the average labor productivities calculated from these values, while Figure 2 (a) through (c) illustrate the production possibility frontiers of Japan, the U.S., and the world, respectively, for this case (Shinkai, 1973). Table 1 (4) indicates how the output of each good in the world changes if Japan specializes in manufacturing and the U.S. specializes in agriculture. This table and Figure 2 (c) reveal that if each country specializes as in Table 1 (4), the world outputs of wheat and cloth increase by 25 $(10^8 t)$ and 10 $(10^8 \, \text{m}^2)$, respectively. If these increased goods are allocated "properly" to each country, the economic welfare of each country is expected to increase.

In this model, the range of π (i.e., the yen/dollar exchange rate), in which mutual trade between Japan and the U.S. may occur, is specified as follows (Ito *et al.*, 1994). Table 1 (5) shows the given wage of each country. Prices of goods are supposed to be decided by the equation "marginal cost = marginal revenue." Under this assumption, as the equation $p = w \cdot dL / dX = w \cdot 1 / q$ (p : price of the goods, w : wage of the country, L : labor input of the sector, X : output of the sector [good], q : labor productivity of the sector) holds for each sector in each country, the price of each good is decided as in Table 1 (6). Therefore, the range of π in which mutual trade may occur between Japan and the U.S. is specified as $62.5 < \pi < 250$ (yen/dollar), as shown in Figure 3.

Table 1. Parameters for Ricardian model

(1) Outputs at present				(2) Allocated labor at present (10^8persons)		
X_o	Japan	U.S.		L_o	Japan	U.S.
Agriculture <Wheat>(10^8t/year)	15	60		Agriculture	0.75	1.20
Manufacturing<Cloth>(10^8m²/year)	10	20		Manufacturing	0.25	0.80
				Total (Endowed)	1.00	2.00

(3) Labor productivity				(4) Gains of the world outputs by specialization		
$q(\equiv X_o/L_o)$	Japan	U.S.		ΔX	Wheat (10^8t/year)	Cloth (10^8m²/year)
Agriculture(t/person/year)	20	50		World outputs at present	75	30
Manufacturing(m²/person/year)	40	25		J→Cloth, U.S.→Wheat Specialization	100	40
				Gains in world outputs	+25	+10

(5) Wage of a worker				(6) Price		
w	Japan	U.S.		p (= w/q)	Japan	U.S.
Agriculture & Manufacturing	100 (10^4yen/person /year)	100 (10^2\$/person /year)		Wheat	5.0 (10^4yen/t)	2.0 (10^2\$/t)
				Cloth	2.5 (10^4yen/m²)	4.0 (10^2\$/m²)
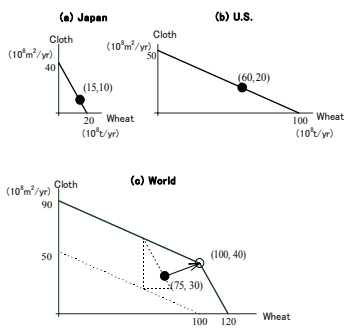
Figure 2. Principle of trade
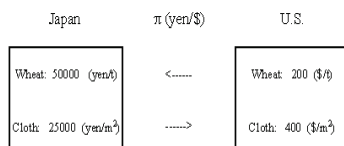
Figure 3. Range of π for mutual trade

3. Three-dimensional Diagrams of the Process toward Trade Equilibrium

By the Ricardian model described above, the outputs of each good in the world increase by the appropriate specialization, and the range of π in which mutual trade may occur is specified. However, by that model, it is not possible to determine the equilibrium amounts of exports and imports or inspect the process leading to trade

equilibrium. Therefore, these problems are considered by the specific factors model, making use of three-dimensional diagrams.

3.1. Study framework

The framework of this study is described below, with k ($k = 1, \cdots, m$) as the country index and i ($i = 1, \cdots, n$) as the sector (good) index (that is common to each country) (Allen, 1967; Henderson *et al*., 1971; Kimura, 2000; Komiya *et al*., 1979).

(1) Production function (for each country concerned; the situation is the same hereafter):

[1] $X_i = A_i \cdot L_i^{\alpha_i}$ $(0 < \alpha_i \leq 1, \quad i = 1, \cdots, n)$

(X_i : output of good i , L_i : labor force of sector i , α_i : coefficient concerning the diminishing returns to scale for sector i , A_i : coefficient concerning the labor productivity for sector i .)

Note that different countries can have different coefficients (e.g., $\alpha_{ki} \neq \alpha_{k'i}$; $A_{ki} \neq A_{k'i}$).

(2) Utility function:

[2] $U = X_1^{'\lambda_1} \cdots X_i^{'\lambda_i} \cdots X_n^{'\lambda_n}$

(X_i' : consumption of good i , λ_i : coefficient concerning the utility of good i)

(3) Restriction by labor force endowment:

[3-1] $L_1 + \cdots\cdots + L_n \leq L_o$

(L_o : endowed labor force of the country; case in which unemployment is permitted.)

[3-2] $L_1 + \cdots\cdots + L_n = L_o$

(Case in which unemployment is not permitted.)

(4) Perfect competitive condition:

[4] $p_i = w \cdot \dfrac{dL_i}{dX_i}$ ($i = 1, \cdots, n$)

(p_i : price of good i by the currency of the country, w : wage of the country [supposed to be an exogenous variable and constant])

(5) Maximization of utility:

[5] $\quad\left(\dfrac{\partial X_j'}{\partial X_i'}\right)_{U=const} = \dfrac{p_i}{p_j}\quad (i, j = 1, \cdots\cdots, n)$

(6) Budget constraint:

[6-1] $\quad p_1(X_1 - X_1') + \quad \cdots \quad + p_n(X_n - X_n') = 0$

[6-2] $\quad \therefore \quad p_1 E_1 + \quad \cdots \quad + p_n E_n = 0$

3.2. Derivation of the export function and other values

The production possibility frontier, supply function, and other values derived from the above equations are described by equations [7] through [12] below.

(1) Production possibility frontier: From equations [1], [3-2]

[7] $\quad\left(\dfrac{X_1}{A_1}\right)^{1/\alpha_1} + \quad \cdots \quad +\left(\dfrac{X_n}{A_n}\right)^{1/\alpha_n} = L_0$

(2) Supply function: From equations [1], [4]

[8] $\quad X_i = a_i \cdot p_i^{\alpha_i/1-\alpha_i}, \left(a_i = \left(\dfrac{\alpha_i A_i^{1/\alpha_i}}{w}\right)^{\alpha_i/1-\alpha_i}, i = 1,\cdots, n\right)$

(3) Demand function for labor: From equations [1], [8]

[9-1] $\quad L_i = \left(\dfrac{\alpha_i A_i}{w}\right)^{1/1-\alpha_i} \cdot p_i^{1/1-\alpha_i} \qquad (i = 1,\cdots, n)$

(l_i : labor demand of sector i)

[9-2] $\quad L = \left(\dfrac{\alpha_1 A_1}{w}\right)^{1/1-\alpha_1} \cdot p_1^{1/1-\alpha_1} + \cdots\cdots + \left(\dfrac{\alpha_n A_n}{w}\right)^{1/1-\alpha_n} \cdot p_n^{1/1-\alpha_n}$

(L : total demand for labor of the country)

(4) "Price possibility frontier"(see next section): From equations [3-2], [9-1]

[10] $\quad\left(\dfrac{\alpha_1 A_1}{w}\right)^{1/1-\alpha_1} \cdot p_1^{1/1-\alpha_1} + \quad \cdots \quad +\left(\dfrac{\alpha_n A_n}{w}\right)^{1/1-\alpha_n} \cdot p_n^{1/1-\alpha_n} = L_0$

(5) Demand function: From equations [2], [5], [6-1], [8]

[11] $\quad X_i' = (\gamma_i a_1 \cdot p_1^{1/1-\alpha_1} + \cdots + \gamma_i a_n \cdot p_n^{1/1-\alpha_n}) \cdot p_i^{-1}, \quad (i = 1,\cdots, n)$
$\qquad$ where $\gamma_i \equiv \lambda_i / (\lambda_1 + \cdots \lambda_n)$
$\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad\qquad$ (Note 2)

(6) Export function (E : + = Export, E : - = Import): From equations [8], [11]

$$[12] \begin{cases} E_1 = X_1 - X_1' = \left((1-\gamma_1)a_1 \cdot p_1^{1/1-\alpha_1} - \cdots \quad - \gamma_1 a_n \cdot p_n^{1/1-\alpha_n} \right) \cdot p_1^{-1} \\ \quad \vdots \\ E_n = X_n - X_n' = \left(\quad - \gamma_n a_1 \cdot p_1^{1/1-\alpha_1} - \cdots + (1-\gamma_n)a_n \cdot p_n^{1/1-\alpha_n} \right) \cdot p_n^{-1} \end{cases} \quad \text{(Note 3)}$$

3.3. Three-dimensional graphs

Examples of the three-dimensional diagrams for equations [7] through [12] are shown below in Figures 4 through 19.   Table 2 shows the parameters of the production and utility functions used for this illustration.

Figures 4 and 5 show the relationships between the indifference curves and amounts of trade and other values for Japan and the U.S, respectively.

Figure 6 shows the offer curves derived from Figures 4 and 5. The equilibrium amount of trade after specialization is denoted by the small circle (○) in Figure 6 (Koizumi *et al.*, 1981; Watanabe, 1991).

Figure 7 shows curves of the demand function for labor with respect to each sector (equation [9-1]) in Japan and the U.S, respectively.

Table 2. Parameters used to illustrate the specific factors model

	Parameter	Japan	U.S.
Agriculture (Wheat: x)	α_1	0.3	0.4
Manufacturing (Cloth: y)	α_2	0.7	0.6
Agriculture (Wheat: x)	λ_1 [1)]	0.5	0.5
Manufacturing (Cloth: y)	λ_2 [2)]	0.07	0.22

1), 2) The condition $\lambda_1/\lambda_2 = \alpha_2/\alpha_1 \cdot L_1^o/L_2^o$ is required so that the production possibility frontier and indifference curve will contact at the present activity point.
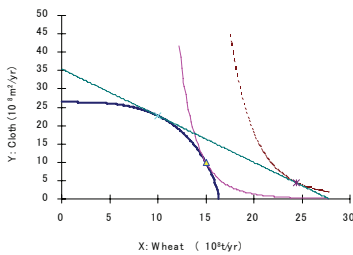

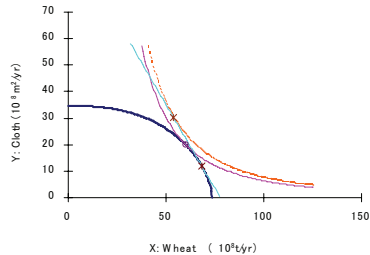Figure 4. Indifference curve and trade - Japan

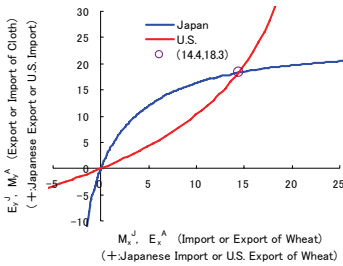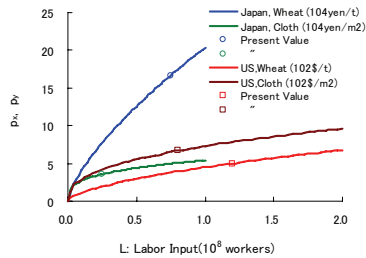Figure 5. Indifference curve and trade - U.S.

Figure 6. Offer curve

Figure 7. Demand function for labor- Japan, U.S.

Figures 8 and 9 show the surfaces of the total demand function for labor (equation [9-2]) with respect to Japan and the U.S, respectively. In Figure 8 the surface within the range of $L < L_o$ (=1.0×10^8 persons) shows the relation between the price set (p_x, p_y) that brings about underemployment and the total demand for labor L. Hereafter, this domain on this surface is called the "domain of underemployment." On the other hand, the horizontal domain on this surface shows the relation between the price set (p_x, p_y) that brings about, at least in one sector, excess profit (i.e., (p_x, p_y) for which equation [4] does not hold) and L. Hereafter, this domain is called the "domain of excess profit." Therefore, the boundary between the domain of underemployment and the domain of excess profit shows the relation between L and the set of prices (p_x, p_y) that allows for both equations [3-2] (the condition for full employment) and equation [4] (perfect competitive condition) to hold. Hereafter, this boundary domain is called the "price possibility frontier," after the "production possibility frontier." The situation is the same as in Figure 9.

Figure 10 shows the price possibility frontier on the $p_x - p_y$ plane for Japan and the U.S. Figure 11 shows this Japanese price possibility frontier as a function of p_x, p_y, prices expressed in dollars, and π. The figure illustrates how the "frontier" varies according to change in π. Note that the price possibility frontier of the U.S. is not affected by π.
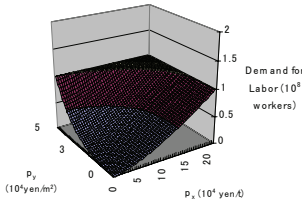
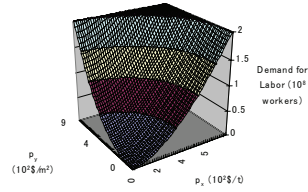Figure 8. Total demand function for labor - Japan
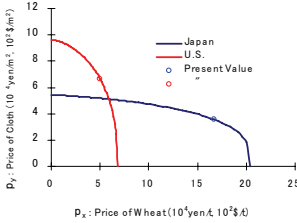

Figure 9. Total demand function for labor - U.S.

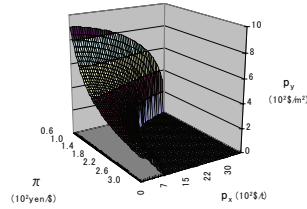Figure 10. Price possibility frontiers - Japan, U.S.

Figure 11. Dependency of price possibility frontiers on π - Japan

Figures 12 and 13 show the supply function (equation [8]) for Japan. However, in this study the supply function is defined on the domain of underemployment or on the price possibility frontier, but not on the domain of excess profit. Likewise, the demand, export, and supply functions in the world market mentioned below are defined using the same domain.

Figures 14 and 15 show the Japanese demand functions (equation [11]) for wheat and cloth, respectively.
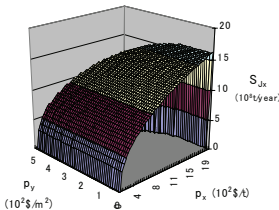

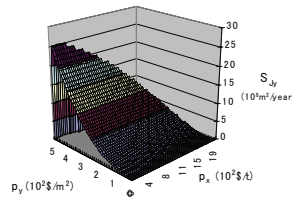Figure 12. Supply function - Japan, wheat, where π =107

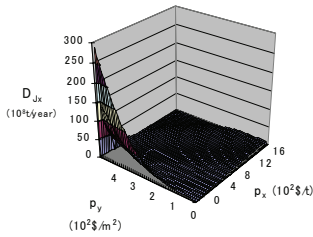

Figure 13. Supply function-Japan, cloth, where π =107

Figure 14. Demand function - Japan, wheat, where π =107
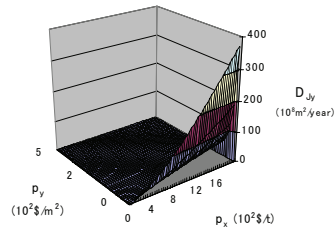
Figure 15. Demand function - Japan, cloth, where π =107

Figures 16 and 17 illustrate the Japanese export functions (equation [12]) for wheat and cloth, respectively, while Figures 18 and 19 show the U.S. export function (equation [12]) for wheat and cloth.
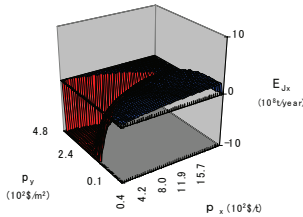
Figure 16. Export function - Japan, wheat, where π =107
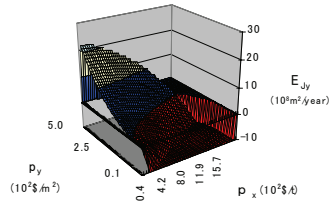
Figure 17. Export function - Japan, cloth, where π =107
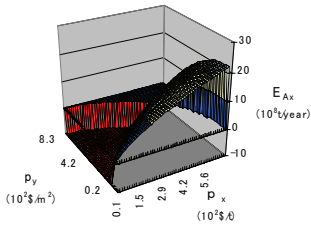
Figure 18. Export function - U.S., wheat
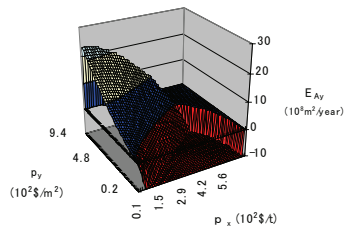
Figure 19. Export function - U.S., cloth

3.4. Deciding the trade equilibrium point based on the price possibility frontier

Next, to construct a more general framework, each function is referred to by a general type. Each function and condition necessary to decide the trade equilibrium point by the general equilibrium model are described in subsections (1) through (8) below.

(1) Supply function (expressed by the currency of country K)

[13]
$$\begin{cases} X_{k1} = X_{k1}(p_{k1}) \\ \quad \vdots \\ X_{kn} = X_{kn}(p_{kn}) \end{cases} \qquad (k = 1, \cdots, m)$$

($X_{k1}, \cdots, X_{kn}$: output of each good in country K. $p_{k1}, \cdots, p_{kn}$: prices of each good expressed by the currency of country K.)

(2) Function for income (expressed by the currency of country K)

[14] $Y_k = Y_k(p_{k1}, \cdots, p_{kn}) = p_{k1} \cdot X_{k1} + \cdots + p_{kn} \cdot X_{kn} \qquad (k = 1, \cdots, m)$

(Y_k : income of country K)

(3) Budget constraint (expressed by the currency of country K)

[15-1] $p_{k1}(X_{k1} - D_{k1}) \ + \quad \cdots \quad + \ p_{kn}(X_{kn} - D_{kn}) \ = 0$
$(k = 1, \cdots, m)$

[15-2] $\therefore \ p_{k1}E_{k1} + \quad \cdots \quad + p_{kn}E_{kn} = 0 \qquad (k = 1, \cdots, m)$

($D_{k1}, \cdots, D_{kn}$: demand for each good in country K)

($E_{k1}, \cdots, E_{kn}$: exports of each good from country K)

(4) Demand function (expressed by the currency of country K)

[16]
$$\begin{cases} D_{k1} = f_{k1}\left\{p_{k1}, \cdots, p_{kn}, \ Y_k(p_{k1}, \cdots, p_{kn})\right\} = D_{k1}(p_{k1}, \cdots, p_{kn}) \\ \quad \vdots \\ D_{kn} = f_{kn}\left\{p_{k1}, \cdots, p_{kn}, \ Y_k(p_{k1}, \cdots, p_{kn})\right\} = D_{kn}(p_{k1}, \cdots, p_{kn}) \end{cases} \quad (k = 1, \cdots, m) \quad \text{(Note 4)}$$

(5) Export function (expressed by the currency of country K)

[17]
$$\begin{cases} E_{k1} = X_{k1} - D_{k1} = E_{k1}(p_{k1}, \cdots, p_{kn}) \\ \quad \vdots \\ E_{kn} = X_{kn} - D_{kn} = E_{kn}(p_{k1}, \cdots, p_{kn}) \end{cases} \qquad (k = 1, \cdots, m)$$

(6) Export function (expressed by international currency [$])

[18]
$$\begin{cases} E_{k1} = E_{k1}(\pi_k p_1, \cdots, \pi_k p_n) \\ \quad \vdots \\ E_{kn} = E_{kn}(\pi_k p_1, \cdots, \pi_k p_n) \end{cases} \qquad (k = 1, \cdots, m)$$

($p_1, \cdots, p_n$: prices expressed by international currency [\$])

π_k : [currency of country K]/[\$] exchange rate, $\pi_m \equiv [\$]/[\$]$ exchange rate = 1)

(7) Condition for the physical balance of supply and demand in the world market

[19]
$$
\begin{cases}
S_{W1} = S_{W1}(\pi_1, \cdots, \pi_n, p_1, \cdots, p_n) \equiv E_{11} + \cdots + E_{m1} = 0 \\
\quad \vdots \\
S_{Wn} = S_{Wn}(\pi_1, \cdots, \pi_n, p_1, \cdots, p_n) \equiv E_{1n} + \cdots + E_{mn} = 0
\end{cases}
$$

($S_{W1}, \cdots, S_{Wn}$: supplies of each good in the world market)

By solving equations [18] and [19], each value of $p_1, \cdots, p_{io-1}, p_{io+1}, \cdots, p_n$ is specified as a function of p_{io} (i_o : good index, arbitrarily chosen), $\pi_1, \cdots, \pi_{m-1}$ (Note 5). If these $p_1, \cdots, p_{io-1}, p_{io+1}, \cdots, p_n$ are substituted into equation [18], the values of $E_{k1}, \cdots, E_{kn}$ ($k = 1, \cdots, m$) are specified as functions of $p_{io}, \pi_1, \cdots, \pi_{m-1}$.

(8) Price possibility frontier

$F_k(p_{k1}, \cdots\cdots, p_{kn}) = 0 \qquad (k = 1, \cdots, m)$

[20] $\qquad \therefore F_k(\pi_k p_1, \cdots\cdots, \pi_k p_n) = 0 \qquad (k = 1, \cdots, m)$

If $p_1, \cdots, p_{io-1}, p_{io+1}, \cdots, p_n$, which have been specified as a function of p_{io} , $\pi_1, \cdots, \pi_{m-1}$ are substituted into equation [20], the equilibrium values of $p_{io}, \pi_1, \cdots, \pi_{m-1}$ are specified. Then, all the values of $\pi_1, \cdots, \pi_{m-1}$, $p_1, \cdots, p_n$ and $E_{k1}, \cdots, E_{kn}$ ($k = 1, \cdots, m$) are specified.

3.5. Specification of the equilibrium point of trade by three-dimensional diagrams

Figures 20 and 21 show the supply functions of wheat or cloth in the world market (left-side member of equation [19]).

Figure 22 illustrates how the trade equilibrium is attained. Curves QR and ST in this figure show the price possibility frontier (equation [20], or more explicitly equation [10]) for Japan and the U.S., respectively, in the case of π=90 (yen/\$). Therefore, the inner domain of OQR shows the domain of underemployment for Japan, as described above. Curve OU represents the Japanese "line of self-sufficient equilibrium" of wheat; that is, the set of (p_x, p_y), which brings about E_{Jx} (export of

wheat from Japan) = 0 in Figure 16. The curve OU also shows Japan's line of self-sufficient equilibrium for cloth; that is, the set of (p_x, p_y), which brings about E_{Jy} (export of cloth from Japan) = 0 in Figure 17. These two lines overlap because, as apparent from equation [15-2]—or more explicitly from equation [6-2]— $E_{k1} = 0$ inevitably leads to $E_{k2} = 0$ in each country in the case of the two-goods model.
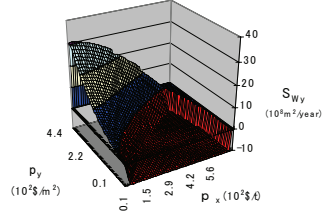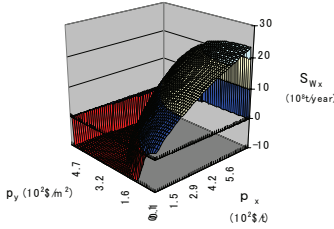

Figure 20. Supply function in the world market - wheat, where $\pi = 107$

Figure 21. Supply function in the world market - cloth, where $\pi = 107$

Domain OUR shows the range where the set of (p_x, p_y) creates $E_{Jx} \geq 0$ (excess exports of wheat from Japan) and at the same time brings about $E_{Jy} \leq 0$ (excess import of cloth to Japan). Similarly, domain OUQ shows the range in which $E_{Jx} \leq 0$ (excess import of wheat to Japan) and $E_{Jy} \geq 0$ (excess export of cloth from Japan) occur. Moreover, domain OVT shows the range in which $E_{Ax} \geq 0$ (excess export of wheat from the U.S.) and $E_{Ay} \leq 0$ (excess import of cloth to the U.S.) occur, and domain OVS shows the range in which $E_{Ax} \leq 0$ (excess import of wheat to the U.S.) and $E_{Ay} \geq 0$ (excess export of cloth from the U.S.) are brought about.

Curve OAB shows the line domain where the set of (p_x, p_y) creates an equilibrium of supply and demand for wheat in the world market; that is, the range where $S_{Wx} = 0$ in Figure 20. At the same time, curve OAB shows the line domain where the equilibrium of supply and demand for cloth in the world market is created; that is, the range where $S_{Wy} = 0$ in Figure 21 is brought about (see Note 5).

Figure 23 illustrates how the curve OAB in Figure 22 varies as the value of π varies. The table for Figure 22 indicates the present and future states of points A, B, and C in Figure 22. This table implies that, under the study's assumption that w_J, w_A are given externally, π varies toward the equilibrium value where the three points A, B, and C coincide and trade equilibrium is attained (Note 6).

Figure 24 shows how the ordinates (i.e., the value of p_y) of points A and B in Figure 22 vary as the value of π varies. As illustrated in this figure, points A, B, and C coincide with each other and trade equilibrium is attained when π = 107 (yen/$).

Figure 25 is the same as Figure 22, except for the case of π = 107 (yen/$), that is, the case of trade equilibrium. Figure 26 is the same as Figure 25, except for the case in which actual data are used, as stated below.
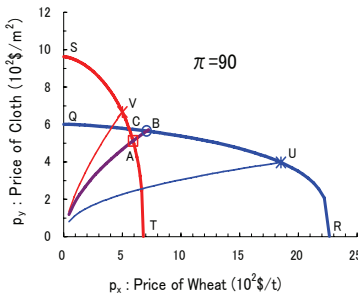
Table for figure 22 (Process leading to trade equilibrium)

Point	Present State	Next (Future) State	
A	Japan: Underemployment	$w_J \downarrow$ or $\pi \uparrow$	$B \rightarrow A$
B	U.S.: Imperfect cpmpeition	$w_A \uparrow$ or $\pi \uparrow$	$A \rightarrow B$
C	World market: Oversupply of cloth & undersupply of Wheat	$p_C \downarrow \& p_W \uparrow \therefore \pi \uparrow$	$C \rightarrow (A \sim B)$

Figure 22. Specification of the trade equilibrium point
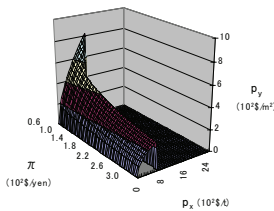
Figure 23. Surface to bring about equilibrium in world trade - common to wheat and cloth
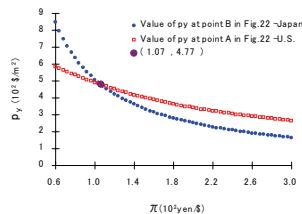
Figure 24. Specification of π (yen/$ rate)
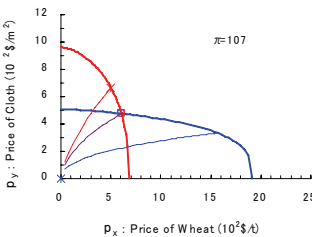
Figure 25. Equilibrium point of trade (result from temporal data)
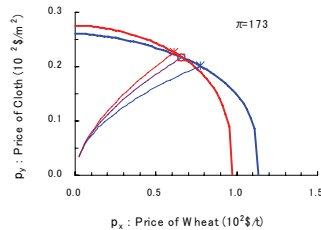
Figure 26. Equilibrium point of trade (result from actual data)

Table 3 (1) – (3) shows the outputs, prices, and the level of utilities in the equilibrium state before and after specialization. Naturally, exports and imports in this table coincide with the one offer curve revealed. This table also illustrates the trade profits.

Table 3. Effects of specialization (result from temporal data)

(1) Effects of specialization on supplies etc.

|		Agriculture (10^8t/yr)			Manufacturing (10^8m^2/yr)		
		Japan	U.S.	World	Japan	U.S.	World
Present Values	S_o	15	60	75	10	20	30
	D_o	15	60	75	10	20	30
	$E_o (\equiv S_o\text{-}D_o)$	0	0	0	0	0	0
Values After Specialization	S'	10.0	68.2	78.2	22.7	12.1	34.8
	D'	24.4	53.8	78.2	4.4	30.4	34.8
	E' ($\equiv$S'-D')	-14.4	14.4	0.0	18.3	-18.3	0.0
Increment	$\Delta S (\equiv S'\text{-}S_o)$	-5.0	8.2	3.2	12.7	-7.9	4.8
	$\Delta D (\equiv D'\text{-}D_o)$	9.4	-6.2	3.2	-5.6	10.4	4.8

(2) Effects of specialization on prices

		Agriculture (unit:t)		Manufacturing (unit:m^2)		Exchange Rate (10^2 yen/\$)
		Japan	U.S.	Japan	U.S.	
Present Values (p_o)	(10^4yen/unit)	16.67	***	3.57	***	***
	(10^2\$/unit)	***	5.00	***	6.67	
Values After Specialization (p')	(10^4yen/unit)	6.46		5.08		1.07
	(10^2\$/unit)	6.06		4.77		
Increasing Rates ($\Delta p/p_o$)		-61%	21%	42%	-29%	***

(3) Effects of specialization on level of utilities

		Japan	U.S.
Present Values	U_o	4.57	15.07
Values After Specialization	U'	5.49	15.67
Increment	$\Delta U/U_o$	20.2%	3.9%

4. Simulation with Actual Data

4.1. Setting up the sectors

Here, Japan and the U.S. are again the focus, and two sectors are considered. The first sector is comprised of four sectors ("agriculture," "forestry," "fishing," and "food"; Note 7), and the second sector is comprised of three sectors ("general machinery," "electric machinery," and "transportation equipment"). The first is set as the sector of Japanese comparative disadvantage (i.e., U.S. comparative advantage). The second is set as the sector of Japanese comparative advantage (i.e., the U.S. comparative disadvantage). The first integrated sector is simply called

"agriculture" and the second is simply called "machinery" hereafter. Simulation is carried out by the two-sector, two-country model using actual data.

Table 4 includes the actual data used, parameter values of the production functions, and wages calculated from actual data (Ministry of Economy, Trade and Industry, 2005; OECD, 2002).

Table 4. Statistical data and estimated parameters

		Japan			U.S.		
		Agriculture[1]	Machinery	Total	Agriculture[1]	Machinery	Total
Statistical Data (2000 year)	Compensation for Employees ($Y_L^{o\,2)}$)	[10^{10}\$/yr]	5.62	24.81	255.72	11.31	35.80	570.18
	Property-type Income (Y_K^o)	"	10.41	11.80	172.95	15.21	17.96	334.28
	Value Added (G_o)	"	20.97	38.70	465.82	29.24	54.97	977.38
	Domestic Products (X_o)	"	46.44	103.46	841.16	76.90	147.68	1687.53
	Total Domestic Demands ($D_o \equiv X_o + M_o - E_o$)	"	51.42	81.90	834.93	77.67	166.50	1729.57
	Exports to the U.S.," to Japan	"	0.07	10.42	15.42	1.32	3.25	8.59
	Exports to ROW [3]	"	0.19	22.79	37.21	3.89	34.55	86.83
	Exports (E_o)	"	0.26	33.21	52.63	5.22	37.80	95.42
	Imports from the U.S.," from Japan	"	1.32	3.25	8.59	0.07	10.42	15.42
	Imports from ROW	"	3.91	8.40	37.81	5.92	46.20	122.04
	Imports (M_o)	"	5.23	11.65	46.40	5.99	56.62	137.46
	Employees (L_o)	[10^8persons]	0.05827	0.04669	0.66610	0.05332	0.06591	1.49805
	Foreign Exchange Rate (π_o)	[yen/\$]			107.77			
Estimated Values	Parameter of Production Function (α) ($\equiv Y_L^o/(Y_L^o + Y_K^o)$)		0.35	0.68	***	0.43	0.67	***
	Parameter of Utility Function (λ)		0.50	0.21	***	0.50	0.40	***
	Wages (w) ($\equiv Y_L^o/L_o$) [4]	[10^2\$/ps/yr]		383.9			380.6	
		[10^4yen/ps/yr]		413.7			410.2	

1) Includes the "forestry", "fishing" and "food" sectors.
2) "o" indicate that the value is the "present" value.
3) ROW; rest of the world.
4) Caluculated using the values in each "Total" column.

4.2. Specification of parameter values

Specification of equilibrium outputs before specialization

Outputs (physical amount) of each good are specified by adopting a "dollar value unit" (abbreviated by "\$vu" hereafter) as the unit of physical output. The "dollar value unit" is defined as the physical amount of goods that could be bought with one dollar (more precisely, by one US dollar in 2000, the year when the above actual data were collected; Niida 1978). When \$vu is adopted, outputs in physical units coincide with outputs in the monetary units in Table 4.

For outputs (i.e., demands) in equilibrium states before specialization, total domestic demands (D_o) in Table 4 are adopted. In general, outputs before specialization should be calculated using the Leontief inverse matrix (Morishima, 1956); however, for convenience, a simpler method is used here.

Specification of α in the production function

If production functions are assumed as in equation [1], relations between Y_L (compensation for employees), Y_K (property-type income; i.e., the producer's surplus), and G_{VD} (gross value added) are expressed as equations [21] and [22] below:

$$p \equiv p(X) = \frac{w}{\alpha A^{1/\alpha}} \cdot X^{1/\alpha - 1} \text{ (From equation [8])}$$

$$G_{VD} = p' \cdot X' = \frac{w}{\alpha A^{1/\alpha}} \cdot X'^{1/\alpha - 1} \cdot X' = \frac{w}{\alpha A^{1/\alpha}} \cdot X'^{1/\alpha}$$

(p' , X' : price or output of each good at any time during the process of specialization)

[21] $\therefore Y_L = \int_0^{X'} p(X)dX = \frac{w}{\alpha A^{1/\alpha}} \cdot \int_0^{X'} X^{1/\alpha - 1} dX = \frac{w}{A^{1/\alpha}} \cdot X'^{1/\alpha} = \alpha \cdot G_{VD}$

[22] $Y_k = G_{VD} - Y_L = (1 - \alpha) \cdot G_{VD}$ (Note 8)

[23] $\therefore \alpha = \frac{Y_L}{Y_k + Y_L}$

Values of α are specified by equation [23].

Specification of λ in utility functions

In the case where the utility function is defined as equation [2], if either λ_1 or λ_2 is specified, then the other is inevitably decided by the relation noted in Table 2. Insofar as this relation holds, the difference of λ_1 , λ_2 has no effect on exports and imports (Table 5(1)) or prices (Table 5(2)) in the equilibrium state, although this difference does affect the level of utility (Table 5(3)). Therefore, a completely arbitrarily value of λ_1 = 0.5 is set in each country.

Specification of w

As mentioned above, w is supposed to be an exogenous variable and constant during the process of specialization. No difference in wages between the sectors in the same country is assumed, and w is calculated as $w \equiv \frac{Y_L^o}{L_o}$ (Y_L^o : total compensation for employees in all sectors in the country at present, L_o : total employees in all sectors in the country). The value in $[10^4 \text{yen/person/year}]$ units

(i.e., 413.7) in Table 4 is adopted for the Japanese wage, and the value in ［10^2\$/person/year］ units (i.e., 380.6) is adopted for the U.S.

4.3. Results

Table 5 (1) – (3) shows the results from the above parameters. As a result of specialization, agriculture outputs decrease by several tens of billions of \$vu and machinery outputs increase by over one hundred billion \$vu in Japan; additionally, the former increases by several tens of billions of \$vu, and the latter decreases by over one hundred billion \$vu in the U.S. This result coincides with the direction of actual trade.

Table 5. Effects of specialization (results from actual data)

(1) Effects of specialization on supplies etc. (unit: 10^{10}\$vu [1])

		Agriculture [2]			Machinery		
		Japan	U.S.	World	Japan	U.S.	World
Present Values	S_o	51.4	77.7	129.1	81.9	166.5	248.4
	D_o	51.4	77.7	129.1	81.9	166.5	248.4
	$E_o (\equiv S_o - D_o)$	0.0	0.0	0.0	0.0	0.0	0.0
Values After Specialization	S'	47.3	82.3	129.6	96.0	153.1	249.1
	D'	55.7	73.9	129.6	70.5	178.7	249.2
	$E' (\equiv S'-D')$	-8.4	8.4	0.0	25.5	-25.6	-0.1
Increments	$\Delta S (\equiv S'-S_o)$	-4.1	4.6	0.5	14.1	-13.4	0.7
	$\Delta D (\equiv D'-D_o)$	4.2	-3.7	0.5	-11.4	12.2	0.8

1) \$vu (=dollar value unit) ≡material amount of goods that can be perchased with one US dollar in the year 2000.
2) Includes the "forestry", "fishing" and "food" sectors.

(2) Effects of specialization on prices

		Agriculture [2]		Machinery		Exchange Rate (10^2
		Japan	U.S.	Japan	U.S.	yen/\$)
Present Values (p_o) [3]	(10^2yen/\$vu)	1.34	***	0.35	***	***
	(\$/\$vu)	***	0.61	***	0.23	
Values After Specialization (p')	(10^2yen/\$vu)	1.15		0.38		1.73
	(\$/\$vu)	0.66		0.22		
Increments ($\Delta p/p_o$)	(%)	-14.2%	8.2%	7.8%	-4.2%	***

3) If the competitive condition (eq.[4]) holds perfectly, values in the upper row of this space will become π_o (=1.0777 ［10^2yen/\$vu］) and values in the lower row will become 1.0000[\$/\$vu].

(3) Effects of specialization on the level of utilities

		Japan	US
Present Values	U_o	17.9	66.8
Values After Specialization	U'	18.0	67.0
Increment	$\Delta U/U_o$	0.9%	0.3%

5. Conclusions

The method presented in this study offers several novel and advantageous features. First, use of this model enables an intuitive understanding of the process leading to the state of trade equilibrium. Second, because this model approaches the Ricardian model when $\alpha \rightarrow 1$, a simple comparison can be made between the two models. Finally, if the production function has the fundamental property of decreasing return to scale, other restrictions on the function can be fairly loose. For example, there is no need to assume the same production function in each country.

The previous model (Ejiri, 2005, Note 9) focused on how evaluation of public benefit from forestry affects the pattern of internal specialization. The main goal of the present study is to clarify the direction of future research—it improves on the previous model by incorporating price into general equilibrium trade theory. The results imply that the method presented here, which reveals the principle of comparative cost theory using three-dimensional diagrams, may be helpful for this purpose. It will also be helpful when analyzing how subsidies for public benefits from forestry or agriculture affect the pattern of international specialization (Ejiri, 1999a, 1999b). In addition, this method will be useful when econometrically analyzing the trade of forest products within a global market (Yukutake *et. al*., 2003, 2006, 2007; Yoshimoto *et al*., 2002), especially when production functions have been set and the proposed model is evaluated in the general equilibrium theory. It could also be useful to evaluate the process of constructing a spatial equilibrium model (Shimamoto, 2002).

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Note 1: The "textiles" sector is excluded because past trade friction between Japan and the U.S. in the textile market should be taken into account. The "petroleum and coal products" sector is excluded because the labor productivity of this sector with respect to value added in Japan (the U.S.) is eight (two) times higher than the average value for all 13 merchandise sectors. These extraordinarily high values probably reflect the prominent capital intensity of this sector, suggesting that it would be difficult to explain the pattern of specialization only by the difference in labor productivity.

Note 2: Equation [11] is derived as follows:
The defining equation for income is as follows:

[24] $\qquad Y = p_1 X_1 + \cdots + p_n X_n$

The next function for income is derived from equations [8] and [24].

[25] $\qquad Y = a_1 \cdot p_1^{1/1-\alpha_1} + \cdots + a_n \cdot p_n^{1/1-\alpha_n}$

Demand function (with explicit variables for income) is derived from equations [2], [5], and [24] as follows:

[26] $\qquad X_i^{'} = \gamma_i \cdot Y \cdot p_i^{-1} \ (\ \gamma_i \equiv \dfrac{\lambda_i}{\lambda_1 + \cdots + \lambda_n}, \ i = 1, \cdots, n \ )$

Therefore, the demand function (with no explicit variables for income) is derived from equations [25] and [26] as equation [11].

Note 3: It is apparent that the budget constraint holds by the next equation, which is derived from equation [12].

[27] $\qquad p_1 E_1 + \cdots + p_n E_n = 0 \ (\because \ \gamma_1 + \cdots + \gamma_n = 1)$

Note 4: Equations [14] and [15] are used to derive equation [16].

Note 5: Under the restriction of equation [15-2], the number of independent equalities in equation [19] is $n - 1$ by Walras's law. The number of unknown variables in equations [18] and [19] are $n + m - 1$ (i.e., $p_1, \cdots, p_n$ and $\pi_1, \cdots, \pi_{m-1}$). Therefore, there remain m unknown variables after solving equations [18] and [19] as follows:

$$\begin{cases} p_1 = f_1(p_{io}, \pi_1, \cdots, \pi_{m-1}) \\ \quad \vdots \\ p_{io-1} = f_{io-1}(p_{io}, \pi_1, \cdots, \pi_{m-1}) \\ p_{io+1} = f_{io+1}(p_{io}, \pi_1, \cdots, \pi_{m-1}) \\ \quad \vdots \\ p_n = f_n(p_{io}, \pi_1, \cdots, \pi_{m-1}) \end{cases}$$

This simultaneous equation expresses a curve with parameters of $\pi_1, \cdots, \pi_{m-1}$ in n-dimensional $\{p_1, \cdots, p_n\}$ space. For example,

1) In the case of two sectors and two countries $\rightarrow p_y = f(p_x, \pi)$. This equation expresses a curve with a parameter π in two-dimensional $\{p_x, p_y\}$ space.

2) In the case of three sectors and four countries $\rightarrow \begin{cases} p_y = f_1(p_x, \pi_1, \pi_2, \pi_3) \\ p_z = f_2(p_x, \pi_1, \pi_2, \pi_3) \end{cases}$. This equation expresses a curve with parameters π_1, π_2, and π_3 in three-dimensional $\{p_x, p_y, p_z\}$ space.

Note 6: If π is supposed to be an exogenous variable, then the rate of wage w_J / w_A is specified.

Note 7: The reasons why the primary industry sector is aggregated with the food product sector are as follows: (1) Many workers in the primary sector work there as a side job, especially in Japan, and therefore it is not appropriate in economic analyses to deal with the primary sector alone; (2) Recently, there has been greater recognition of the importance of policies that integrate the primary and food product sectors.

Note 8: In this study, producers (entrepreneurs) are assumed to consume all producer surpluses, as specified by equation [22], and to buy final goods according to the same principle as for workers' consumption. Namely, the producer is assumed to consume goods based on the same utility function as the worker. In this case, the sum of producer and worker consumptions becomes $Y_K + Y_L = (1 - \alpha) \cdot G_{VA} + \alpha \cdot G_{VA} = G_{VA}$. Summing up each side member of this equation with respect to all sectors leads to the "left-side member $= G_{DE}$ (gross domestic expenditure)" and "right-side member $= G_{DP}$ (gross domestic product)." This means that the relation "gross domestic expenditure = gross domestic product" falls under this assumption at any time during the specialization process.

Note 9: In this study, the production function was limited to the Leontief type; thus, inputs for intermediate goods were fixed automatically, according to the output for each type of good. Therefore, when incorporating price factor, only equilibrium in trade of final goods should be assumed to be fixed by supply export and demand import curves. As for the forest products, the curves estimated by Yukutake *et al*. (2003) and Yukutake *et al*. (2006) are applicable.

3次元図表による比較生産費モデルの考察

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要約: 各国の供給関数, 輸出関数, および世界市場における各財の供給関数等を 3 次元図表化し, これを用いて貿易均衡点を特定する手法を考案した. 貿易モデルは多数国多数財を対象にした一般均衡型のものを想定したが, 実際の数値計算は日米 2 国の 2 財モデルについて行った. 貨幣賃金は外生変数とし, 生産関数は投入労働力のみを変数とするとする特殊要素モデルとした. まず, 国内の各産業が投入要素として需要する労働力の合計値たる労働力総需要を各財の価格の関数として導き, この労働力総需要関数を 3 次元図表として表示した. 次に, この労働力総需要関数および賦存労働力の完全雇用条件から定まる曲線を, 財価格空間内における "価格可能フロンティア" と定義し, この曲線を特定した. さらに, この "価格可能フロンティア" と世界市場における各財の需給均衡条件等により, 貿易均衡点を特定した. 最後に, 2000年日米国際産業連関表等のデータを当モデルに適用し, 貿易の均衡点等を特定した. 試算結果は, 現実の貿易とほぼ同一の方向に貿易が進行することなどを示しており, 当モデルが, 一般均衡論の中で農林業の外部経済効果の内部化を図る際に, その判定材料の一つになりうることが確認できた.

キーワード: 比較生産費, 特殊要素モデル, 特化パターン, 貿易均衡, 3 次元図表