Analysis on the Reverted Mean of Log Price Dynamics Through Stochastic Modeling

Atsushi Yoshimoto

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Abstract: Expectation of the future trend and volatility of log prices has an impact on forest management decisions under stochastic environments. In this paper, we investigate the reverted mean level of log price dynamics using several variants of the mean-reverting process. Market-based log price data in Fukuoka Prefecture, along with nationwide averages, are used for the analysis. Target timber products are 3 and 4 m sugi (*Cryptomeria japonica*) and hinoki (*Chamercypress obtusa*) dimensional log. Parameter estimation is carried out by a quasi-maximum likelihood method. Our analysis shows that the reverted mean price differs significantly between the nationwide average and market-based price dynamics. Because the nationwide average price data smoothes market-based price dynamics, the underlying volatility is underestimated when compared to market-based price data. The estimated parameter values of all models from the nationwide average price are also much smaller than the estimated parameters from the market-based models. The reverted mean derived from the market-based data tends to show a decreasing trend over the time horizon, while the nationwide data does not.

1. Introduction

Many researchers have highlighted the importance of considering the stochastic nature of management decisions. Among them are Reed (1984), Brazee and Mendelsohn (1988), Clarke and Reed (1989), Clarke and Reed (1990), Teeter and Caulfield (1991), Zinkhan (1991), Thomson (1992), Reed and Ye (1994), Yin and Newman (1995), Yin and Newman (1996), Yoshimoto and Shoji (1998), Willassen

(1998), Plantinga (1998), Morck *et al*. (1989), Thorsen (1999), Brazee and Bulte (2000), Fina *et al*. (2000), Hughes (2000), Yoshimoto (2002), Sødal (2003), Insley (2002), Insley and Rollins (2005), and Penttinen (2006). Depending upon the initial assumptions, several candidate models can be proposed when constructing a stochastic model for timber price dynamics. In recent literature on stochastic modeling for price dynamics, stochastic differential equations are often utilized where the drift and volatility terms are some function of time and state. Different models can be created by changing these functional forms (see Duffie, 1992).

Geometric Brownian motion is widely applied as a stochastic model because of the ease of model handling. Clarke and Reed (1989), Thomson (1992), Yoshimoto and Shoji (1998), and Yoshimoto (2002) support the use of geometric Brownian motion for price dynamics, whereas Haight and Holmes (1991) support a non-stationary random walk for the log-transformed price process. Insley (2002) and Insley and Rollins (2005) support the geometric mean-reverting process because of microeconomic reasons related to the long-run marginal cost of production reflecting the mean price. Yoshimoto and Shoji (2002) tested 13 different state-dependent volatility models for 13 time series data of log price. They point out the importance of preparing candidate models before finalizing an appropriate model for target price dynamics.

Expectation of the future trend and volatility of log prices has an impact on forest management decisions under stochastic environments. It is therefore important to capture these dynamics through a "reasonable" stochastic model for the purpose of forecasting log price over time. The objective of this paper is to investigate the future trend of two different types of log price dynamics through several variants of the mean-reverting process. One type is the market-based log price data in Fukuoka Prefecture; the other is nationwide average data. Most analyses of log price changes have been based on nationwide average data, while local forest owners are subject to nearby auction market prices. Because the aggregated data has less information on price changes than the market data, it is necessary to investigate their difference. In our analysis, we focus on the reverted mean of the price dynamics using four of the thirteen stochastic models with the mean-reverting property proposed by Yoshimoto and Shoji (2002). The paper is organized as follows. In the next section, the four stochastic models used here are described using the parameter estimation method. In the third section, the estimated parameters, and the reverted mean of each price dynamic, are provided as the results of our analysis. The final section presents some concluding remarks.

2. State Dependent Volatility Models and Parameter Estimation

In this paper, we use the state dependent volatility process with the mean-reverting property for log price dynamics (denoted as SDVP-MR hereafter). Letting x_t be a log price at time t, SDVP-MR is expressed by,

[1] Case 1: $dx_t = (\alpha - \beta x_t)dt + x_t^{\gamma}\sigma dB_t$

where B_t is a standard Brownian motion with the following characteristics:

1. $B_0 = 0$
2. $\{B_t, t \geq 0\}$ has stationary and independent change
3. for all t (>0), B_t follows the normal distribution with a variance of t and a mean of 0

The set of parameters (α, β, σ) are positive coefficients and thus strictly constrain equation [1] to be mean-reverting. Changing the value of γ to 1, ½, and 0, as in Yoshimoto and Shoji (2002), three other mean-reverting models can be considered:

[2] Case 2: $dx_t = (\alpha - \beta x_t)dt + x_t\sigma dB_t$

[3] Case 3: $dx_t = (\alpha - \beta x_t)dt + \sqrt{x_t}\sigma dB_t$

[4] Case 4: $dx_t = (\alpha - \beta x_t)dt + \sigma dB_t$

With the exception of model [4], parameter estimation is carried out by the pseudo-likelihood approach based on discretization by the local linearization method. The idea of the local linearization method was first introduced by Ozaki (1985) (see also Ozaki, 1992, 1993), and has been utilized for parameter estimation of nonlinear stochastic models with a finite sample. Shoji and Ozaki (1997) and Shoji (1998) implemented an extension of this method. An original nonlinear stochastic differential equation is first converted into a stochastic differential equation with a constant diffusion term, and then a nonlinear drift term of the derived stochastic differential equation is locally approximated by a linear function of the state and time over a small time elapse. Because the resultant stochastic differential equation can be solved analytically, the corresponding likelihood function for parameter estimation is derived. A detailed description of this method can be also found in Yoshimoto and Shoji (2002).

Following Yoshimoto and Shoji (2002), for SDVP-MR

[5] $$dx_t = (\alpha - \beta x_t)dt + x_t^{\gamma}\sigma dB_t \ ,$$

the quasi log-likelihood function for the data set, $\{ x_t \}$, under $\gamma \neq 1, 0$ is derived as

[6] $$\log(p(x_{t_0}, x_{t_1}, \cdots, x_{t_N})) = -\frac{1}{2}\sum_{i=1}^{N}\{\frac{(y_{t_i} - E_{t_{i-1}})^2}{V_{t_{i-1}}} + \log(2\pi V_{t_{i-1}})\} + \log(p(y_{t_0})) + \sum_{i=0}^{N}\log\{\frac{d\phi(x_{t_i})}{dx}\}$$

where

[7] $$E_{t_i} = y_{t_i} + \frac{h(y_{t_i})}{L_{t_i}}(e^{L_{t_i}\Delta t} - 1) + \frac{M_{t_i}}{L_{t_i}^2}(e^{L_{t_i}\Delta t} - 1 - L_{t_i}\Delta t)$$

[8] $$V_{t_i} = \frac{\sigma^2 \cdot (\exp(2L_{t_i}\Delta t) - 1)}{2L_{t_i}}$$

Elements in the above two equations [7] and [8] are:

[9] $$y_{t_i} = \phi(x_{t_i}) = \frac{1}{1-\gamma}x_{t_i}^{1-r}$$

[10] $$h(y_{t_i}) = \alpha x_{t_i}^{-\gamma} - \beta x_{t_i}^{-\gamma+1} - \frac{\gamma}{2}x_{t_i}^{\gamma-1}\sigma^2$$

[11] $$L_{t_i} = -\alpha\gamma x_{t_i}^{-1} - (1-\gamma)\beta - \frac{\gamma(\gamma-1)}{2}\sigma^2 x_{t_i}^{2\gamma-2}$$

[12] $$M_{t_i} = \frac{1}{2}\sigma^2\{\alpha\gamma x_{t_i}^{\gamma-2} - \gamma(\gamma-1)^2\sigma^2 x_{t_i}^{3\gamma-3}\}$$

For $\gamma = 1$, we have

[13] $$y_{t_i} = \phi(x_{t_i}) = \ln(x_{t_i})$$

[14] $$h(y_{t_i}) = \frac{1}{x_{t_i}} \cdot (\alpha - \beta x_{t_i}) - \frac{1}{2}\sigma^2$$

[15] $$L_{t_i} = -\frac{\alpha}{x_{t_i}}$$

[16] $$M_{t_i} = \frac{\alpha\sigma^2}{2x_{t_i}^2}.$$

For $\gamma = 0$, we have an analytical solution, so that the log-likelihood function is obtained

[17] $$\log(p(x_{t_0}, x_{t_1}, \cdots, x_{t_N})) = -\frac{1}{2}\sum_{i=1}^{N}\{\frac{(x_{t_i} - E_{t_{i-1}})^2}{V_{t_{i-1}}} + \log(2\pi V_{t_{i-1}})\} + \log(p(x_{t_0}))$$

where

[18] $$E_{t_i} = x_{t_i}e^{-\beta\Delta t} + \frac{\alpha}{\beta}(e^{-\beta\Delta t} - 1)$$

[19] $$V_{t_i} = (\sigma^2(1 - e^{-2\beta\Delta t})/2\beta$$

Note that t_n is the time of the *n*-th observation and y_{t_n} is the corresponding log-transformed data of x_{t_n}. In order to strictly constrain parameters (α, β, σ) to be positive, the following exponential transformation is applied where $(\theta_1, \theta_2, \theta_3)$ are unrestricted in the range of $(-\infty, \infty)$.

[20] $$\alpha = e^{\theta_1}, \beta = e^{\theta_2}, \sigma = e^{\theta_3}$$

All parameters are estimated by maximizing the corresponding log-likelihood function.

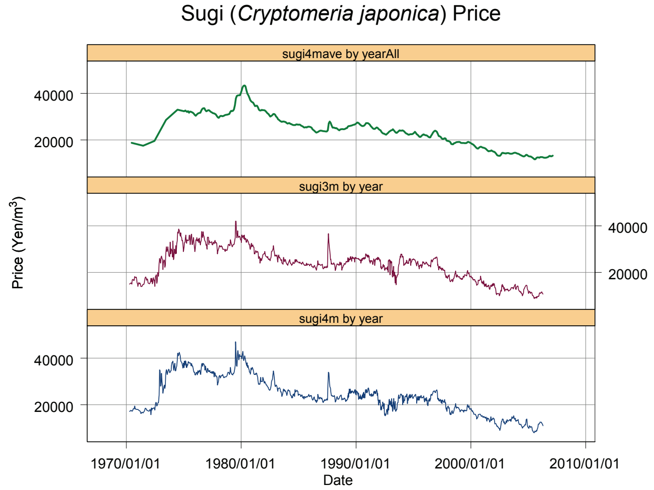
3. Analysis of Estimated Parameters and Reverted Means

Analysis was performed for two types of the time series data, nationwide average and market-based log prices for sugi (*Cryptomeria japonica*) and hinoki (*Chamaecypress obtusa*). Nationwide price data was taken from monthly time series data of log prices between January 1975 and September 2006, along with annual data from 1970 to 1975, to develop 392 data points covering the duration from 1970 to 2006. This data was taken from the Japanese Forestry Agency's annual report on the supply and demand of timber (Rinyacho, 1975-2006). Logs included in the report were offered for sale in the Japanese timber market and had a diameter of 14-22cm and a length of 3.65-4.0m. The market-based data was taken from the log market at Fukuoka Prefecture (Ukiha Log Auction Market) and included 985 data points. Logs included in this data had a diameter of 14-18cm and lengths of 3 or 4m. Figure 1 shows price dynamics of these data sets from 1970 to 2006. Sugi3m, sugi4m, and sugi4mave represent sugi with 3 and 4m length market-based data and 4m nationwide average data, respectively. Likewise, hinoki3m, hinoki4m, and hinoki4mave represent the same corresponding data points for hinoki logs. An increasing price trend can be observed for the first decade, with a peak in the late 1980s, followed by a decreasing trend to the present time (see Figure 1).

We analyzed seven different data sets to estimate the parameters. Yoshimoto and Kato (2004) observed that analysis with geometric Brownian motion may result in varying parameter estimates, depending upon the duration of estimation for the target time series data. With the use of SDVP-MR, we investigated how the estimated parameters change with respect to the corresponding available data over different periods. We started estimation with the complete data set first, and then reduced the duration by 5 years from the beginning up to the year 2000 as the last starting period. In other words, the first estimation used data from April of 1970 to 2006, the second used data from April of 1975 to 2006, and so on, until the last estimation used the data from April of 2000 to 2006.

Figures 2 and 3 show the results of parameter estimation over the different starting dates for sugi and hinoki, respectively, along with statistics of the parameter estimates for different log products and different models in Table 1. Note that only the estimated value of σ was presented in the logarithmic scale in Figures 2-d and 3-d. As for sugi log price, the value of α in Figure 2-a approached zero, ranging from 0 to 0.4 for sugi4mave. The value of sugi4m and sugi3m was much larger than that of sugi4mave for all models, ranging from 0.02 to 1.24 for sugi4m and from 0.13 to 1.49 for sugi3m. Comparing the estimated values across different data sets, a relatively large value was observed for the last set of the time series data from April 2000 to the end. As for the estimated value of β in Figure 2-b, the changing trend over different starting points was similar to that of α. Again, a relatively small value for sugi4mave was observed, as opposed to those for sugi3m and sugi4m. The estimated value ranged from 0 to 0.58 for sugi4mave, from 0 to 1.64 for sugi4m, and from 0.22 to 2.00 for sugi3m.

a)



b)

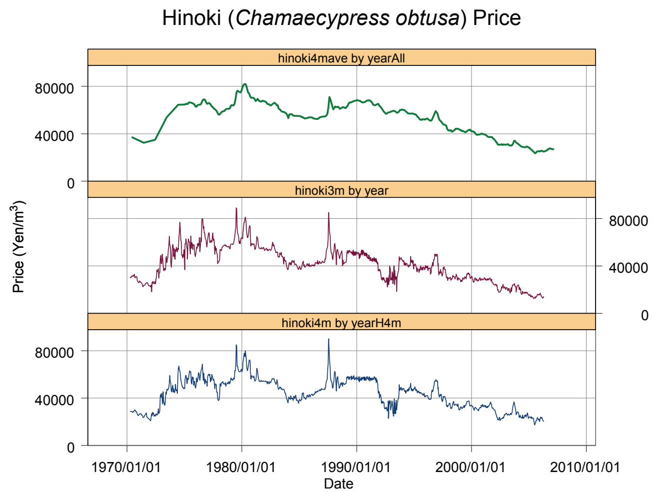


Figure 1. Price dynamics

a) sugi (*Cryptomeria japonica*) b) hinoki (*Chamaecypress obtusa*)

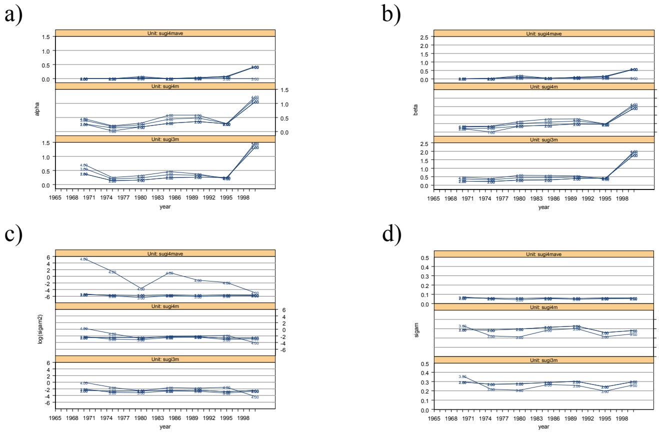


Figure 2. Estimated parameter values for sugi price

a) α b) β c) Log-transformed σ^2 d) σ

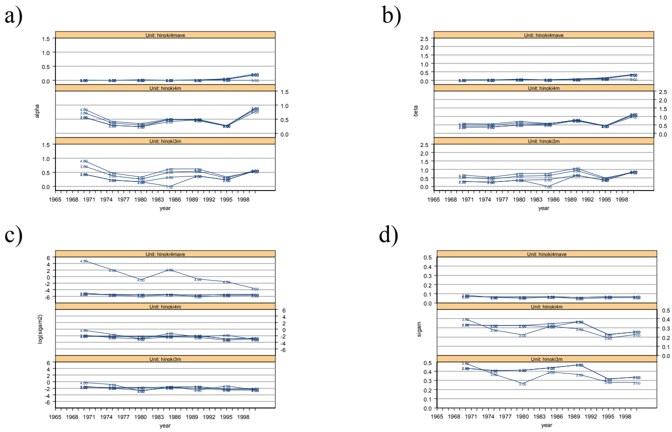


Figure 3. Estimated parameter values for hinoki price

a) α b) β c) Log-transformed σ^2 d) σ

Table 1. Statistics of parameter estimates for different log products

		Case 1			Case 2			Case 3			Case 4		
		α	β	σ^2	α	β	σ^2	α	β	σ^2	α	β	σ^2
sugi3m	Min	0.13372	0.21642	0.05933	0.13373	0.21645	0.05932	0.20020	0.32226	0.03905	0.20629	0.35596	0.01390
	1st Qu.	0.20231	0.27068	0.07487	0.20233	0.27071	0.07486	0.23897	0.38937	0.04553	0.28622	0.43212	0.12289
	Mean	0.39415	0.52526	0.08031	0.39418	0.52530	0.08030	0.48073	0.63057	0.06702	0.54531	0.70468	0.26550
	Median	0.25095	0.30991	0.08485	0.25095	0.30995	0.08484	0.33052	0.44406	0.06705	0.37222	0.55505	0.20152
	3rd Qu.	0.32256	0.41013	0.08829	0.32259	0.41014	0.08829	0.46395	0.47191	0.07104	0.58829	0.57672	0.21540
	Max	1.32468	1.78891	0.09164	1.32471	1.78895	0.09163	1.42853	1.92512	0.12986	1.48960	2.00405	0.96649
	ý9tal N	7	7	7	7	7	7	7	7	7	7	7	7
	Std.Dev.	0.41816	0.56224	0.01141	0.41817	0.56224	0.01141	0.43606	0.57362	0.03082	0.44886	0.57967	0.31849
sugi4m	Min	0.01965	0.00017	0.06636	0.13149	0.19317	0.06635	0.18415	0.29583	0.04472	0.20462	0.33935	0.01736
	1st Qu.	0.21722	0.26979	0.07997	0.21723	0.28653	0.08007	0.26446	0.39754	0.04738	0.29118	0.40731	0.09273
	Mean	0.34763	0.46702	0.08651	0.36363	0.49939	0.08653	0.45812	0.62539	0.06893	0.52259	0.70784	0.30615
	Median	0.27180	0.39171	0.08371	0.27184	0.39175	0.08371	0.40089	0.50448	0.06006	0.46135	0.62070	0.13178
	3rd Qu.	0.32616	0.46673	0.09350	0.32619	0.46676	0.09349	0.46337	0.61647	0.08634	0.58524	0.76886	0.21232
	Max	1.05519	1.40424	0.10854	1.05522	1.40428	0.10854	1.16615	1.54942	0.11029	1.23933	1.64252	1.38380
	Total N	7	7	7	7	7	7	7	7	7	7	7	7
	Std.Dev.	0.33078	0.44570	0.01387	0.31461	0.41321	0.01386	0.33087	0.42748	0.02568	0.34970	0.44950	0.48165
sugi4mave	Min	0.00000	0.00746	0.00321	0.00000	0.00748	0.00321	0.00000	0.00622	0.00167	0.00000	0.00591	0.00708
	1st Qu.	0.00000	0.02735	0.00337	0.00184	0.02985	0.00337	0.00000	0.02646	0.00245	0.00000	0.02545	0.08902
	Mean	0.08095	0.14082	0.00352	0.08147	0.14153	0.00352	0.00000	0.03469	0.00288	0.08013	0.14285	31.65556
	Median	0.02854	0.08971	0.00358	0.02854	0.08971	0.00358	0.00000	0.04257	0.00271	0.02055	0.07173	0.30845
	3rd Qu.	0.05609	0.12833	0.00360	0.05609	0.12833	0.00360	0.00000	0.04552	0.00286	0.06278	0.15406	3.71826
	Max	0.42590	0.57717	0.00394	0.42590	0.57717	0.00394	0.00000	0.05009	0.00518	0.41482	0.56326	213.65881
	Total N	7	7	7	7	7	7	7	7	7	7	7	7
	Std.Dev.	0.15477	0.19924	0.00024	0.15445	0.19878	0.00024	0.00000	0.01576	0.00110	0.15033	0.19619	80.27474
hinoki3m	Min	0.00011	0.00031	0.10040	0.15897	0.24003	0.10039	0.25561	0.43258	0.07113	0.33100	0.50101	0.05914
	1st Qu.	0.19014	0.26624	0.13839	0.22308	0.32251	0.13837	0.33460	0.49129	0.07768	0.40402	0.60546	0.07843
	Mean	0.28011	0.39410	0.16295	0.32659	0.45085	0.16320	0.46359	0.63568	0.12577	0.54403	0.73184	0.27241
	Median	0.22477	0.35239	0.16806	0.32530	0.36788	0.16802	0.50278	0.60059	0.13200	0.52427	0.75816	0.23064
	3rd Qu.	0.40029	0.50747	0.18746	0.40035	0.52221	0.18840	0.54751	0.75192	0.14307	0.61647	0.79239	0.35000
	Max	0.55502	0.85856	0.22049	0.55503	0.85857	0.22046	0.72249	0.93016	0.23574	0.91197	1.06800	0.76022
	Total N	7	7	7	7	7	7	7	7	7	7	7	7
	Std.Dev.	0.18520	0.28027	0.04295	0.13804	0.22126	0.04315	0.16470	0.19601	0.05847	0.19995	0.18988	0.25365
hinoki4m	Min	0.23589	0.36971	0.05163	0.23591	0.36975	0.05163	0.27086	0.44039	0.03590	0.25294	0.41574	0.04356
	1st Qu.	0.27462	0.40568	0.08205	0.27464	0.40571	0.08577	0.32737	0.48823	0.05133	0.37947	0.56278	0.07567
	Mean	0.45994	0.59228	0.09625	0.44898	0.57949	0.09929	0.49779	0.63719	0.07917	0.51407	0.65029	0.22516
	Median	0.47771	0.49148	0.10533	0.41886	0.47735	0.10613	0.48397	0.55457	0.07640	0.45194	0.58188	0.15601
	3rd Qu.	0.53421	0.66869	0.10874	0.52526	0.63094	0.11526	0.61740	0.70565	0.09155	0.63860	0.71865	0.24317
	Max	0.88832	1.13603	0.13522	0.88830	1.13601	0.13520	0.84017	1.07764	0.15611	0.85748	0.99154	0.73888
	Total N	7	7	7	7	7	7	7	7	7	7	7	7
	Std.Dev.	0.22979	0.27663	0.02835	0.22963	0.28004	0.02966	0.21900	0.22770	0.04047	0.22126	0.18331	0.24177
hinoki4mave	Min	0.00000	0.00615	0.00352	0.00000	0.00615	0.00352	0.00000	0.00519	0.00211	0.00000	0.00891	0.02644
	1st Qu.	0.00000	0.02756	0.00422	0.00000	0.02756	0.00422	0.00000	0.02402	0.00288	0.00000	0.02128	0.30769
	Mean	0.04120	0.09564	0.00433	0.04120	0.09564	0.00433	0.00000	0.03894	0.00369	0.03442	0.08388	21.78649
	Median	0.00183	0.04256	0.00444	0.00183	0.04256	0.00444	0.00000	0.04007	0.00339	0.00530	0.05333	0.48583
	3rd Qu.	0.03330	0.10667	0.00451	0.03330	0.10667	0.00451	0.00000	0.05737	0.00381	0.02414	0.08651	7.97045
	Max	0.21994	0.35230	0.00487	0.21994	0.35230	0.00487	0.00000	0.06458	0.00697	0.18737	0.30932	135.43689
	Total N	7	7	7	7	7	7	7	7	7	7	7	7
	Std.Dev.	0.08112	0.12121	0.00042	0.08112	0.12121	0.00042	0.00000	0.02219	0.00158	0.06847	0.10492	50.24636

The log-transformed value of σ^2 changed largely for the Case 4 model due to its constant volatility term, as observed in Figure 2-c. The estimated non-transformed value of σ in the Case 4 model had a negative correlation of -0.23 and -0.25 with those of α and β, while it was 0.43, 0.45, and 0.60 with α and 0.39, 0.41, and 0.47 with β for Cases 1, 2, and 3, respectively. Figure 2-d shows the comparison of the estimated values, σ, with the result of Case 4 removed. The largest difference between log products was observed for σ. The value for sugi4mave ranged from 0.041 to 0.072, with a mean of 0.060, while it ranged from 0.21 to 0.33 with a mean of 0.28 for sugi4m and from 0.20 to 0.36 with a mean of 0.27 for sugi3m. The aggregated data was much less volatile than the market-based data.

In the case of hinoki, the same tendency was observed as in sugi. Hinoki4mave had much smaller values of (α, β, σ) than the other two products, hinoki4m and hinoki3m. Only the value of σ for Case 4 was quite large. Comparing the results of hinoki with those of sugi, 51 out of 84 cases had a larger α value for hinoki than sugi, while 43 cases had a larger value for β. For σ, the number of cases increased to 66, implying that the price of hinoki logs is more volatile than sugi.

Based on the estimated parameter values, the reverted mean was calculated as follows: First, the target stochastic differential equation was converted into an equation with a constant volatility process

[21] $$y_t = \phi(x_t)$$

which satisfies

[22] $$\phi'(x_t) \cdot g(x_t) = 1$$

and $g(x_t)$ is the state dependent part of the volatility term,

[23] $$g(x_t) = \begin{cases} x_t^{\gamma} & \text{for Case 1} \\ x_t & \text{for Case 2} \\ x_t^{0.5} & \text{for Case 3} \\ 1 & \text{for Case 4} \end{cases}$$

With this conversion, we have the following constant volatility process:

[24] $$dy_t = \{\phi'(x_t)(\alpha - \beta x_t) + \frac{1}{2}\phi'' g(x_t)^2 \sigma^2\}dt + \sigma dB_t$$

Setting the drift term equal to zero, we estimate the reverted mean from the following equation:

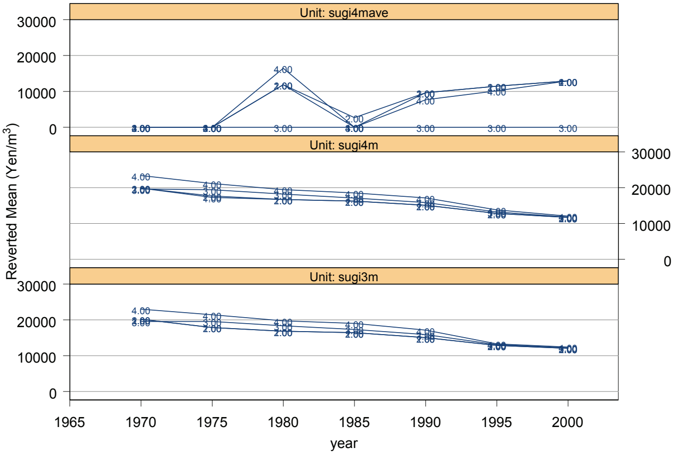
[25] $$\phi'(x_t)(\alpha - \beta x_t) + \frac{1}{2}\phi'' g(x_t)^2 \sigma^2 = 0$$

Figure 4 depicts the reverted mean derived for different log products and different models over different periods of time, while Table 2 shows their statistics. As observed in Figure 4, the nationwide average log data tended to result in a lower reverted mean than the others. When the Case 3 model was used, the derived mean was very close to zero. Since 1990, most showed an increasing trend in value over different starting dates of the time series data for estimation. This could be due to the small degree of concavity in the price change. On the other hand, in the case of the market-based price, except for one hinoki3m point in the Case 1 model, all showed a decreasing trend. Sugi4m decreased from 23,384 Yen/m^3 as a maximum to 11,696 Yen/m^3 as a minimum, while sugi3m showed a similar decline of 23,000 Yen/m^3 to 12,065 Yen/m^3. Hinoki demonstrated higher value than sugi. Hinoki4m decreased from 43,090 Yen/m^3 to 24,466 Yen/m^3, and hinoki3m decreased from 40,743 Yen/m^3 to 17,479 Yen/m^3, except at one point. All things considered, a decreasing trend in price change was crucial for the current time series data set under the market-based price, while it was not so for the nationwide average price.

4. Concluding Remarks

From an economic viewpoint on price movement, a mean-reverting stochastic model would be preferable because of the microeconomic reasons for the long-run marginal cost of production reflecting the mean price (Insley 2002). We do not expect price to increase infinitely or decrease to zero or lower over the time horizon. In this paper, we applied a state dependent volatility process with a mean-reverting property to investigate how the reverted mean differs when price dynamics of different products and models are applied. Our analysis demonstrated that reverted mean price varies depending on the models and products used. There was a significant difference between the nationwide average and market-based price dynamics. Because the nationwide average price data smoothes the market-based price dynamics, the underlying volatility is underestimated when compared to the

a)



b)

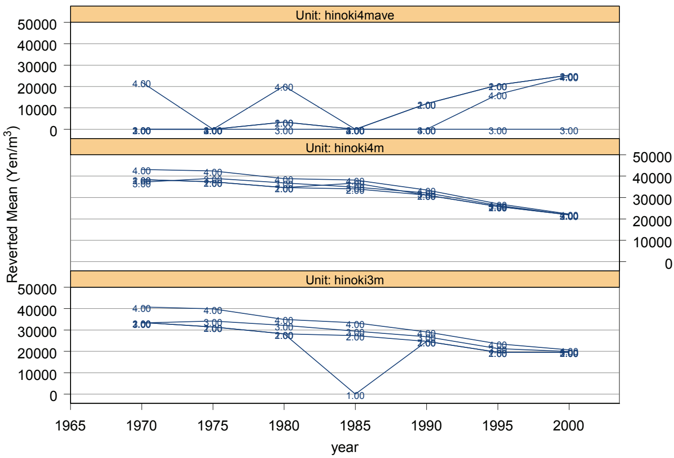


Figure 4. Reverted mean for different price dynamics
a) sugi (*Cryptomeria japonica*) b) hinoki (*Chamaecypress obtusa*)

market-based price data. The estimated parameter values of all models from the nationwide average price are also much smaller than the estimated parameters from the market-based models. This implies that using the nationwide average data could result in the wrong reverted mean for forecasting the future trend of price dynamics for local forest owners. Forest owners must face market-based prices; thus, it may be important to focus analysis on local markets. There is an urgent need for the analysis of local market data because almost all research on log prices uses nationwide average data that is frequently more available. Forestry practices in local forest areas are increasingly diminished by unfavorable price changes.

Table 2. Statistics of the derived reverted mean by different models and products

		Log Products					
		sugi4m	sugi3m	sugi4mave	hinoki4m	hinoki3m	hinoki4mave
Case 1	Min	11,696	12,065	2	24,466	46	3
	Mean	15,690	15,894	6,564	32,630	22,107	8,764
	Max	19,894	20,187	12,947	38,381	33,484	25,309
	Std.Dev.	2,776	2,831	6,214	5,648	11,351	10,677
Case 2	Min	11,696	12,065	0	24,466	17,479	0
	Mean	15,745	15,894	6,946	32,246	26,011	8,761
	Max	19,894	20,187	12,947	38,382	33,484	25,309
	Std.Dev.	2,818	2,831	5,813	5,412	5,880	10,680
Case 3	Min	11,813	12,171	0	24,508	17,839	0
	Mean	16,477	16,558	0	32,984	27,852	0
	Max	19,646	19,551	0	39,019	34,167	0
	Std.Dev.	3,025	2,986	0	5,622	6,257	0
Case 4	Min	12,073	12,413	0	24,944	18,449	0
	Mean	17,946	18,018	6,760	35,444	31,435	11,902
	Max	23,384	23,000	16,471	43,090	40,743	24,715
	Std.Dev.	3,976	3,972	6,861	7,186	8,279	11,407
Overall	Min	11,696	12,065	0	24,466	46	0
	Mean	16,464	16,591	5,068	33,326	26,851	7,357
	Max	23,384	23,000	16,471	43,090	40,743	25,309
	Std.Dev	3,144	3,136	5,954	5,807	8,484	10,000

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確率モデリングによる木材価格の平均回帰値分析

吉本　敦

要約：将来的な価格の平均的な動向とその変動は森林管理の意思決定に大きな影響を与える. 本論文では, 平均回帰を示す確率モデルを用いて木材価格の平均回帰値の分析を行った. 個々で使用したデータは全国平均価格に加え, 福岡県の木材市場価格でり, 3 m, 4 mのスギ, ヒノキ材である. 確率モデルのパラメータ推定には擬最尤法を採用した. 分析の結果, 平均回帰値は全国平均価格と市場価格には大きな乖離があることが分かった. 全国平均価格は市場価格を平均して得られるため, その結果ボラティリティが市場価格のものと比較して過少になる. また, 推定されたパラメータ値も全国平均価格の方がかなり小さくなった. 平均回帰値については, 市場価格では時間の経過に伴い減少傾向を示したが, 全国平均ではそのような傾向は観察されなかった.

キーワード：林分管理，木材価格，確率微分方程式，確率動的計画法，確率モデル